TG 1168
JUNE 1971
Copy No. 3

Technical Memorandum

TROPOSPHERIC RANGE ERROR AT THE ZENITH

HELEN S. HOPFIELD

THE JOHNS HOPKINS UNIVERSITY ■ APPLIED PHYSICS LABORATORY

35

R

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) The Johns Hopkins University Applied Physics Lab. 8621 Georgia Avenue Silver Spring, Maryland 20910	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP NA

3. REPORT TITLE

Tropospheric Range Error at the Zenith

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Helen S. Hopfield

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1971	37	11

8a. CONTRACT OR GRANT NO. N00017-62-C-0604	9a. ORIGINATOR'S REPORT NUMBER(S) TG 1168
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	National Aeronautics and Space Admin.

13. ABSTRACT

The atmospheric range error in an electromagnetic signal arriving vertically at the earth is measured by the height integral of the refractivity N through the atmosphere ($N = 10^6 (n-1)$ where n is the index of refraction). N in the non-ionized atmosphere is the sum of "dry" and "wet" components, which are considered separately. Only the dry part is important at optical wavelengths and the present discussion deals principally with this dry part. Data from several thousand meteorological balloons were studied to develop a method for predicting the height integrals of N_{dry}. It is found theoretically and from observed data that, although N_{dry} depends on both temperature and pressure, its height integral is a linear function of surface pressure only. The slope is nearly the same, worldwide; there is, however, a very small but detectable latitude variation, corresponding to the latitude variation of g. It is shown that the vertical range error (dry component) at a given latitude can be predicted within 2 rms from surface pressure alone. Irregularities in the temperature profile do not affect the prediction. In view of these results, the equivalent height for a quartic N_{dry} model must vary as surface temperature; it is approximately 40 km at 0°C and becomes zero near 0°K at all locations. Possible small differences between parameters for different stations are being examined.

14. KEY WORDS

Electromagnetic signal refractivity
Meteorological balloon information
Temperature profiles
Tropospheric range error

TG 1168
JUNE 1971

Technical Memorandum

TROPOSPHERIC RANGE ERROR AT THE ZENITH

HELEN S. HOPFIELD

THE JOHNS HOPKINS UNIVERSITY ▪ APPLIED PHYSICS LABORATORY
8621 Georgia Avenue, Silver Spring, Maryland 20910
Operating under Contract N00017-62-C-0604 with the Department of the Navy

211a

ABSTRACT

The atmospheric range error in an electromagnetic signal arriving vertically at the earth is measured by the height integral of the refractivity N through the atmosphere. ($N \equiv 10^6 (n-1)$ where n is the index of refraction.) N in the non-ionized atmosphere is the sum of "dry" and "wet" components, which are considered separately. Only the dry part is important at optical wavelengths, and the present discussion deals principally with this dry part. Data from several thousand meteorological balloons were studied to develop a method for predicting the height integrals of N_{dry}. It is found theoretically and from observed data that, although N_{dry} depends on both temperature and pressure, its height integral is a linear function of surface pressure only. The slope is nearly the same, worldwide; there is, however, a very small but detectable latitude variation, corresponding to the latitude variation of g. It is shown that the vertical range error (dry component) at a given latitude can be predicted within 2 mm rms from surface pressure alone. Irregularities in the temperature profile do not affect the prediction in the zenith direction. In view of these results, the equivalent height for a quartic N_{dry} model must vary as surface temperature; it is approximately 40 km at 0°C and becomes zero near 0°K at all locations. Possible small differences between parameters for different stations are being examined

CONTENTS

Preceding page blank

ILLUSTRATIONS

Preceding page blank

1. INTRODUCTION

The electromagnetically measured range from the earth to an object outside the atmosphere, e.g., the moon or an artificial satellite, is too large by more than 2 meters if the object is directly overhead, and by about twice that amount if the zenith angle of the line of sight is 60°. The precise amount of the atmospheric effect on range is given by the integral of $(n-1)$ along the signal path, where n is the index of refraction. For convenience, the refractivity N is defined as $10^6 (n-1)$. Since N depends on atmospheric conditions, it varies along the signal path.

In the optical region, N for air is also a function of wavelength, and range measurements at two optical wavelengths could be used to remove refraction effects from the data. The instrumentation requirements for this are, however, very severe. In the radio region, the refractivity of the uncharged atmosphere (troposphere and stratosphere) is independent of signal frequency up to 15 000 MHz.

A theoretical refraction correction, if adequate, would be simpler than multifrequency measurements. A model to provide this correction was developed for radio use (initially for use with satellite doppler data), and the so-called "dry" part of it can also be applied in the optical region with little change.

2. THEORY

In order to predict the atmospheric range error in an electromagnetic signal at any elevation angle, it is necessary to know the range error for a signal in the zenith direction, i.e., the height integral $10^{-6}\int N dh$ in the existing state of the atmosphere. The error at a small zenith angle can be obtained directly from this, being little affected by the height profile of N. For very large zenith angles, however, it is necessary to know the N profile also.

ZENITH INTEGRAL OF N

The refractivity of air at radio frequencies up to 15 000 MHz can be expressed as the sum of two components, "dry" and "wet," which will be designated by the subscripts d and w, respectively. Their values at a given point of the atmosphere can be computed from meteorological data. The equations given by Smith and Weintraub (Ref. 1) are:

$$\left.\begin{aligned} N_d &= \frac{77.6\ P}{T} \\ N_w &= \frac{3.73 \times 10^5 e}{T^2}, \end{aligned}\right\} \qquad (1)$$

in which T is the temperature in degrees Kelvin, P is the total pressure in millibars, and e is the partial pressure of water vapor, also in millibars.

At optical frequencies, the effect of water vapor is so much decreased as to be practically negligible. The above expression for N_d, however, is in close agreement with values of N given by the Edlén expression (Ref. 2) for the phase refractivity of dry air in the infrared (wavelengths of a few microns). The following discussion of the height integral of N_d therefore pertains either to the dry component of radio range error or to the entire optical range error in

Preceding page blank

the near infrared; but in the optical case, a dispersion correction of a few percent will be needed for laser wavelengths in the visible (Ref. 3).

The height integral of N_d in Eq. (1) can be deduced from the gas laws without knowledge of the shape of the N_d profile. If V is the volume and R is the gas constant for unit mass of dry air, if we assume that air is a perfect gas, we have, from the gas laws,

$$PV = RT$$

$$\frac{P}{T} = \frac{R}{V} = R\rho \, , \tag{2}$$

where ρ is the density. The combination of Eqs. (1) and (2) yields:

$$N_d = 10^{-3} \times 77.6 \, R\rho \, ,$$

if R and ρ are in cgs units. Then

$$\int N_d \, dh = 10^{-3} \times 77.6 \, R \int \rho \, dh \, . \tag{3}$$

The pressure at the surface is:

$$P_s = \int \rho \, g \, dh \, . \tag{4}$$

If g, the acceleration of gravity, is assumed constant over the height range of the lower atmosphere, the combination of Eqs. (3) and (4) yields an atmospheric height contribution of:

$$\Delta h = \int (n-1) dh = 10^{-9} \times \frac{77.6 \, R \, P_s}{g} \, , \tag{5}$$

if all quantities are in cgs units. Thus theoretically, in a dry atmosphere, the zenith range error produced by refraction is directly proportional to the surface pressure and is independent of the way in which temperature varies with height (Refs. 4 and 5). Observed data to test this conclusion for the real atmosphere will be presented below.

The above theory does not apply to the wet component, which will be discussed briefly later.

HEIGHT PROFILE OF N

Although the N profile is not needed to predict the atmospheric effect at or near the zenith, this is not true at low elevation angles. To be theoretically valid, a profile model must account for a zenith integral of N which is consistent with Eq. (5). In developing a model it is assumed that each component of N (dry or wet) is a function of its surface value and of height above the earth, but not of horizontal position or time during a short series of observations. Thus diurnal variations and weather fronts are neglected for the time being. The dry and wet components are different functions of height.

In a dry atmosphere, at heights small enough so that the acceleration of gravity g can be considered constant, the pressure (Ref. 6) is given by:

$$P = P_s e^{-\frac{g}{R}\int_o^h \frac{dh}{T}} , \tag{6}$$

where P_s is the pressure at the surface of the earth, R is the gas constant, and h is height above the surface.

If the temperature T does not vary with height in the atmosphere, this becomes:

$$P = P_s e^{-\frac{gh}{RT_s}} . \tag{7}$$

Combining Eq. (7) with Eq. (1) yields an exponential N profile for a dry isothermal atmosphere:

$$N = N_s e^{-\frac{gh}{RT_s}} . \qquad (8)$$

Exponential models for the total N and for its separate components have been extensively used (Refs. 7, 8, and 9).

In the earth's atmosphere, however, the temperature on the average decreases with height at a fairly constant lapse rate in the troposphere, is moderately constant in the tropopause region, and above that increases slowly with height in the stratosphere. Considering the troposphere first, let us assume that the lapse rate α is constant $\left(\alpha \equiv -\frac{dT}{dh}\right)$. Then:

$$T = T_s - \alpha h . \qquad (9)$$

Substituting Eq. (9) into Eq. (6) and integrating, we get (Ref. 6):

$$P = P_s \left(\frac{T_s - \alpha h}{T_s}\right) . \qquad (10)$$

Thus, the pressure when $\alpha \neq 0$ is not an exponential function of height. If P and T from Eqs. (9) and (10) are used in Eq. (1), the N_d profile can be put in the form (Ref. 4):

$$N_d = N_{d_s} \left[\frac{\frac{T_s}{\alpha} - h}{\frac{T_s}{\alpha}}\right]^{\mu} , \qquad (11)$$

where

$$\mu = \frac{g}{R\alpha} - 1 . \qquad (12)$$

The degree of the N_d profile is shown in Fig. 1 as a function of the temperature lapse rate α. For any positive value of α (negative $\frac{dT}{dh}$) which is likely to occur in the troposphere, μ is positive.

The degree μ would be zero for a temperature lapse rate of 34°C/km, an improbably high value. The adiabatic lapse rate is 9.8°C/km (Ref. 6), but observed tropospheric lapse rates are generally less (~ 7°C/km in warm climates, less near the poles). α and μ are negative in the region of a temperature inversion, and negative in the stratosphere above the tropopause.

It was shown earlier (Ref. 5) that the zenith integral of N_d in an atmosphere with constant lapse rate (zero, positive or negative), as obtained by integrating Eq. (8) or Eq. (11) with appropriate limits, is identical to the value of Eq. (5). If the lapse rate is constant and known, it is therefore possible to write an equation for the N_d profile which will match an observed profile as regards both the zenith integral and the profile shape. Since in practice the lapse rate is only fairly constant in the troposphere and changes sign at the tropopause, any single mathematical function will not match the actual N_d profile shape perfectly at all heights. A single function can, however, yield the correct zenith integral, regardless of irregularities in the profile shape; and also provide a usable approximation to the profile shape in the denser part of the atmosphere.

A fourth-degree model ($\mu = 4$ in Eqs. (11) and (12)), which was developed earlier (Ref. 4), corresponds, if $g = 980\ \mathrm{cm/s^2}$, to a temperature lapse rate of 6.828°C/km, and does on the average match observed profiles well to a considerable height, in regions of the earth where this lapse rate is a realistic value in the troposphere (Fig. 2).

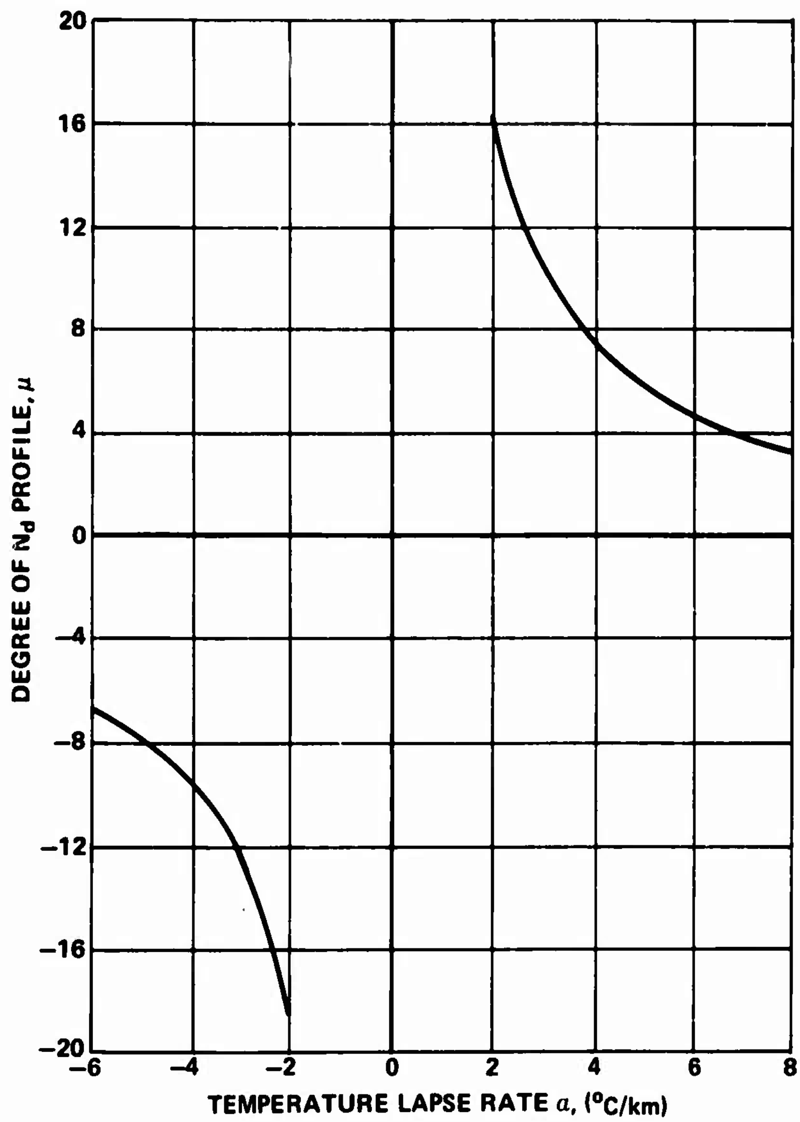

Fig. 1 DEGREE OF THEORETICAL N_d PROFILE

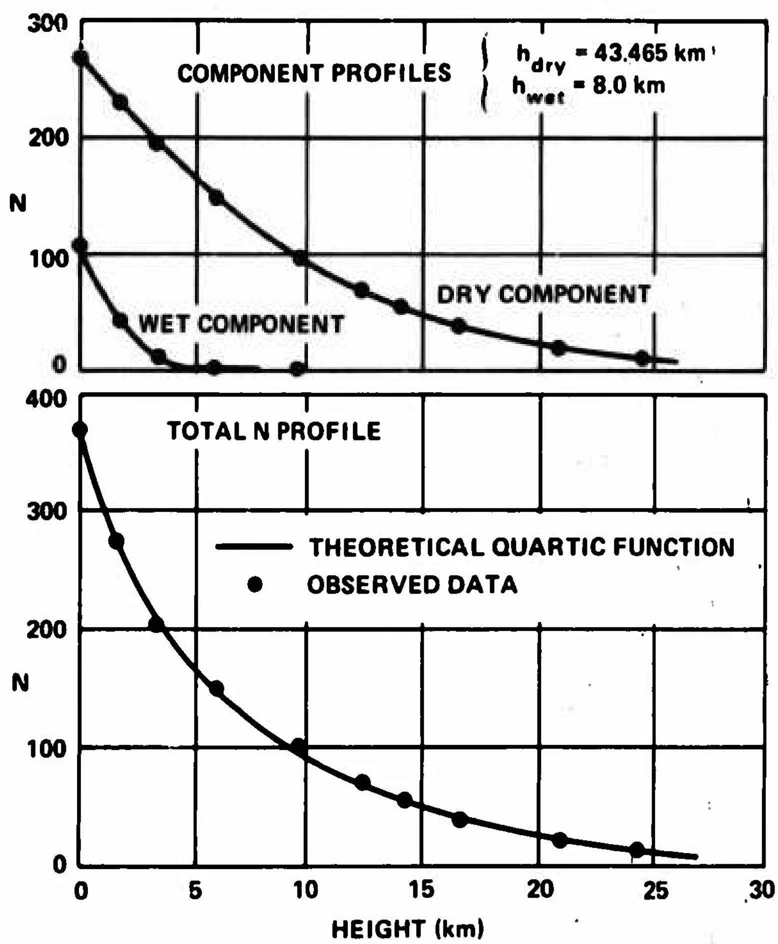

Fig. 2 PROFILE OF TROPOSPHERIC REFRACTIVITY N, WEATHER SHIP E, 35°N, 48° W, JULY 1967

The quartic model provides an integrable expression for the range correction $\int N \, d\rho$ along the slant range vector ρ at any elevation angle, and this can be used as a range correction at elevation angles for which the bending of the signal path is negligible. An algorithm for simplifying the computation of the correction has been developed by Yionoulis (Ref. 10).

Height parameters for the quartic model have been obtained from observed data that will be described below. Let us write the quartic profile expression in the form:

$$N_d = \frac{N_{d_s}}{h_d^4} (h_d - h)^4 , \tag{13}$$

where the height h_d replaces T_s/α of Eq. (11). For $T_s = 273.16°K$ (0°C) and $\alpha = 6.828°/km$, $T_s/\alpha = 40.006$ km; but h_d is now to be determined empirically. Heights are to be measured above the surface. The zenith range effect is now:

$$\Delta h = 10^{-6} \int N_d \, dh = 10^{-6} \cdot \frac{N_{d_s} h_d}{5} , \tag{14}$$

in which h_d and the height error Δh are in the same units. In view of Eq. (1), this can be compatible with Eq. (5) only if h_d is proportional to T_K. This hypothesis was tested by examining balloon meteorological data, as will be described in the next section.

Height parameters for the wet component N_W for the radio case were also obtained from the data. The theoretical considerations given above do not apply to the wet term, but for convenience, it has been assumed that a quartic profile can be used for N_W also. The work on the wet component will be mentioned only briefly; it is less far advanced, and also is not of interest for laser applications. Except at very low angles, the dry component accounts for at least 90% of the range effect even at radio frequencies.

The wet component, however, is responsible for most of the variability of the radio range effect and is much less predictable than the dry part. Much more work is needed on the wet component.

3. DATA AND COMPUTATIONS

Meteorological balloon data in 14 one-year sets, from ten different geographical locations, have been studied. There were two balloons per day in each case: so far, a total of approximately 10 000 balloons. For each balloon flight, data were obtained at the standard pressure levels and also at additional "significant levels," i.e., generally 50 or 60 observed points per balloon ascent. The data at each point included height, pressure, temperature, and relative humidity, thus providing the necessary information for computing the dry and wet components of N from Eq. (1) (for radio frequencies). Each balloon yielded one observed profile of N. The data were on magnetic tape and the computations were done with an IBM 360/91 computer.

The zenith integrals of N_d and N_w were obtained from these N profiles by numerical integration. Occasional flights in which any sort of equipment failure occurred were deleted from the data collection. The data used included only flights which gave complete dry data to at least the 30-mb level (~ 24 km) and complete wet data to at least the 500-mb level (5.5 to 6 km). In computing the numerical integrals, a small correction was added at the top so as to approximate the true total integral. For the dry component, this correction was based on the topmost observed pressure and Eq. (5) (a slight improvement over the correction used in Ref. 5).

Two one-year sets of the zenith integrals obtained from these balloon data are shown in Figs. 3 and 4: one for Washington, D. C. (85 meters above sea level), and one for Albuquerque, New Mexico (1620 meters above sea level). The three sections of these figures show, respectively, the total integral and the separate integrals of the wet and the dry components, as functions of time during the year. Each point represents data from a single balloon flight.

It is noticeable that the dry component integral shows little seasonal variation, although some weather effects can

Preceding page blank

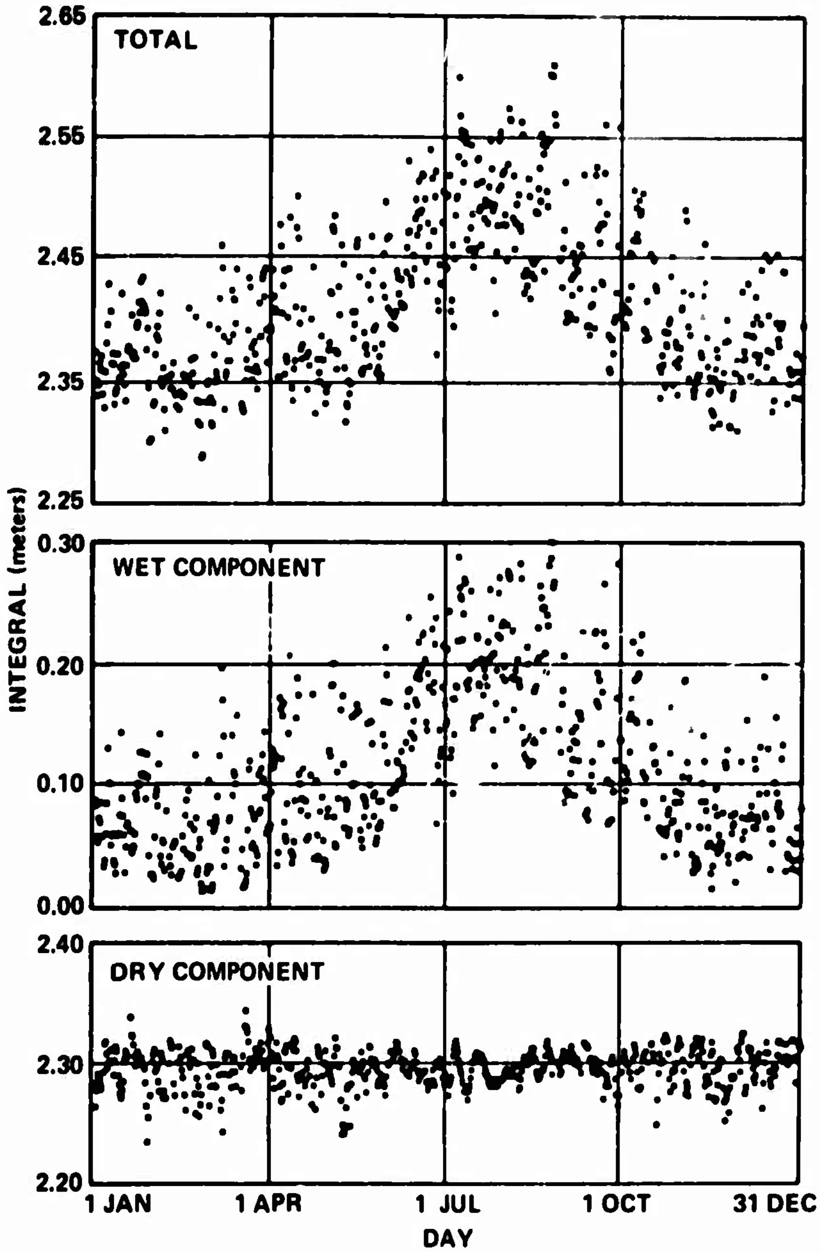

Fig. 3 VERTICAL INTEGRAL OF REFRACTIVITY DURING A YEAR, BALLOON DATA, WASHINGTON, D. C., U. S. A. – 1967

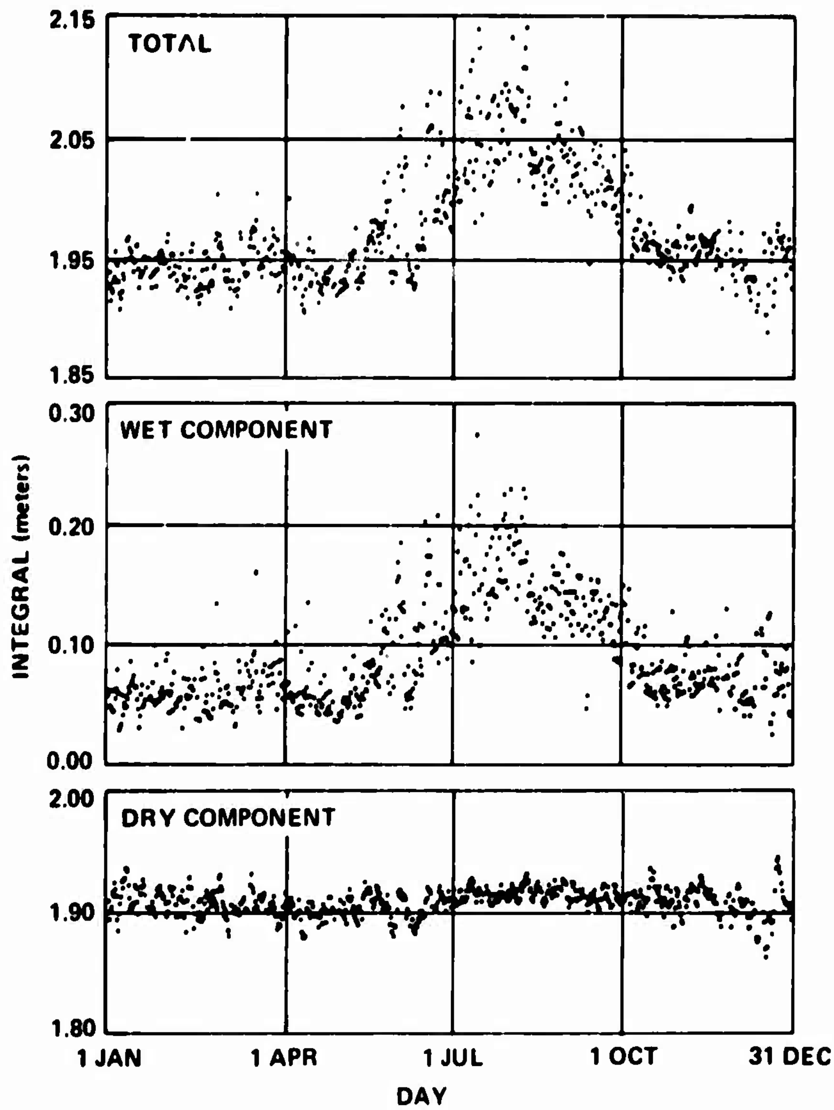

Fig. 4 VERTICAL INTEGRAL OF REFRACTIVITY DURING A YEAR, BALLOON DATA, ALBUQUERQUE, NEW MEXICO, U. S. A. – 1967

be seen. The wet component shows large seasonal and also weather effects.

The wet integral $\int N_w\, dh$ is less than $\int N_d\, dh$ by an order of magnitude or more, yet produces most of the variation in the total integral, at radio frequencies. At laser wavelengths, the wet contribution is so much reduced as to be practically negligible.

The average value of $\int N_d\, dh$ through the atmosphere, for the year, was approximately 2.3 meters at Washington, while that at Albuquerque was 40 cm less, or 1.9 meters, because of the different heights of the two stations above sea level. The average radio value of $\int N_w\, dh$ for the year at Albuquerque was approximately 60% of that at Washington.

4. RESULTS

$\int N_d\, dh$ VERSUS SURFACE PRESSURE

Figure 5 shows samples of the height integral of N_d as a function of surface pressure for several samples of data from different locations. Equation (5) predicts a linear relation. The plotted points are observed values of the integral from the balloon data. It was assumed (cf. Eq. (5)) that:

$$\int N_d\, dh = k P_s \ , \tag{15}$$

and the slope k was obtained from each one-year set of observed data by a least squares procedure. The results are listed in Table 1, and the lines given by Eq. (15) with these empirical slopes k are shown respectively for the samples of Fig. 5. For each one-year data set, the rms error in predicting $\int N_d\, dh$ from the empirical value of k was between 1 and 2 mm. The theoretical line of Eq. (5) was also obtained for each case, using for each a mean value of g at mid-atmosphere (6 km high, approximately the 500 mb level), at the latitude of the station (Ref. 11). The theoretical and empirical lines are so nearly coincident that they cannot be shown as separate on the scale of the figure.

The delaying effect of the dry atmosphere (troposphere and stratosphere) on a ranging signal in the zenith direction can therefore be predicted with an rms deviation of the order of 2 mm of range (one-way travel of the signal).

The empirical slopes for the different locations are nearly but not quite the same. The theoretical slope (Eq. (5)) is a function of the local value of g, hence it varies slightly with latitude. Figure 6 shows this variation. The curve represents the theoretical slope as a function of latitude, using for g its value at the local latitude at a height of 6 km above sea level. The separate points on the

Table 1

Observed Slope k for relation $\int N_d \, dh = k \, P_s$

Station	Latitude	Longitude	Height (meters)	Year	$\int N_d \, dh$ Mean, (meters)	k (m/mb)	Prediction error in $\int N_d \, dh$ σ(meters)
Weather Ship E	35°N	48°W	10	1963	2.32085	0.002279048	0.00152
Weather Ship E	35°N	48°W	10	1965	2.31787	0.002278113	0.00161
Weather Ship E	35°N	48°W	10	1967	2.32390	0.002277716	0.00178
Ascension Island	7°55'S	14°24'W	79	1967	2.29241	0.002282242	0.00088
Caribou, Maine	46°52'N	68°01'W	191	1967	2.25882	0.002276762	0.00190
Washington, D.C. (Dulles Airport)	38°59'N	77°28'W	85	1967	2.29636	0.002277612	0.00200
St. Cloud, Minn.	45°35'N	94°11'W	318	1967	2.22478	0.002276946	0.00150
Columbia, Mo.	38°58'N	92°22'W	239	1967	2.25030	0.002277840	0.00189
Albuquerque, New Mexico	35°03'N	106°37'W	1620	1967	1.90810	0.002276673	0.00146
Vandenberg Air Force Base, California	34°44'N	120°34'W	100	1967	2.28651	0.002277228	0.00150
Wake Island	19°17'N	166°39'E	5	1963	2.30960	0.002280307	0.00159
Wake Island	19°17'N	166°39'E	5	1965	2.31234	0.002280026	0.00160
Wake Island	19°17'N	166°39'E	5	1967	2.31013	0.002280070	0.00174
Pago Pago, Samoa	14°20'S	170°43'W	5	1967	2.30569	0.002280862	0.00171

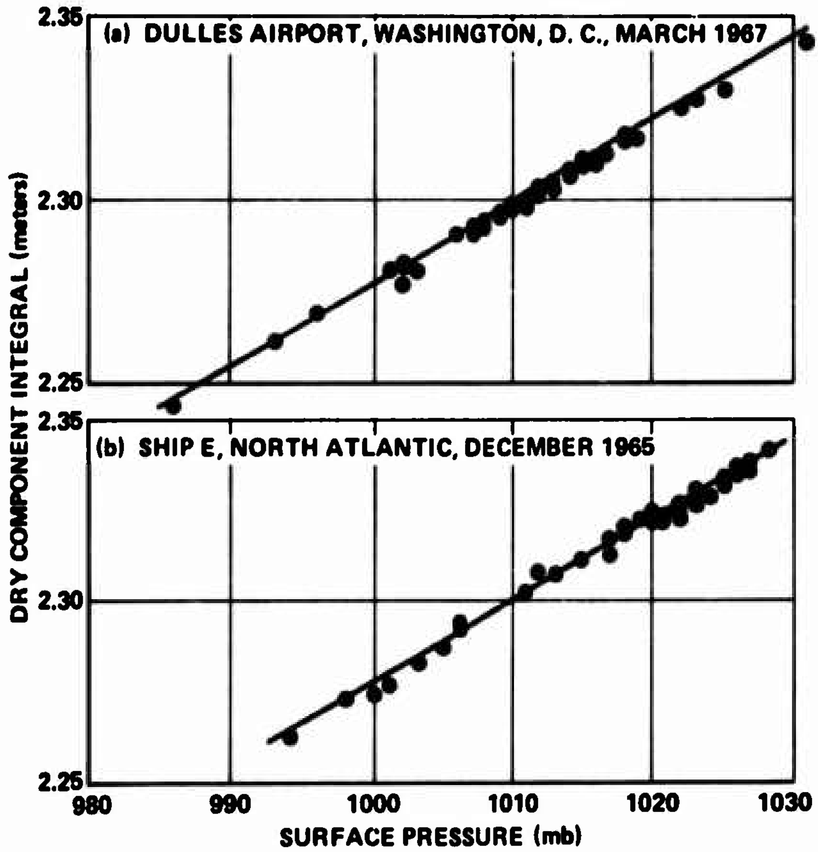

Fig. 5 VERTICAL INTEGRAL OF DRY COMPONENT OF REFRACTIVITY VERSUS SURFACE PRESSURE

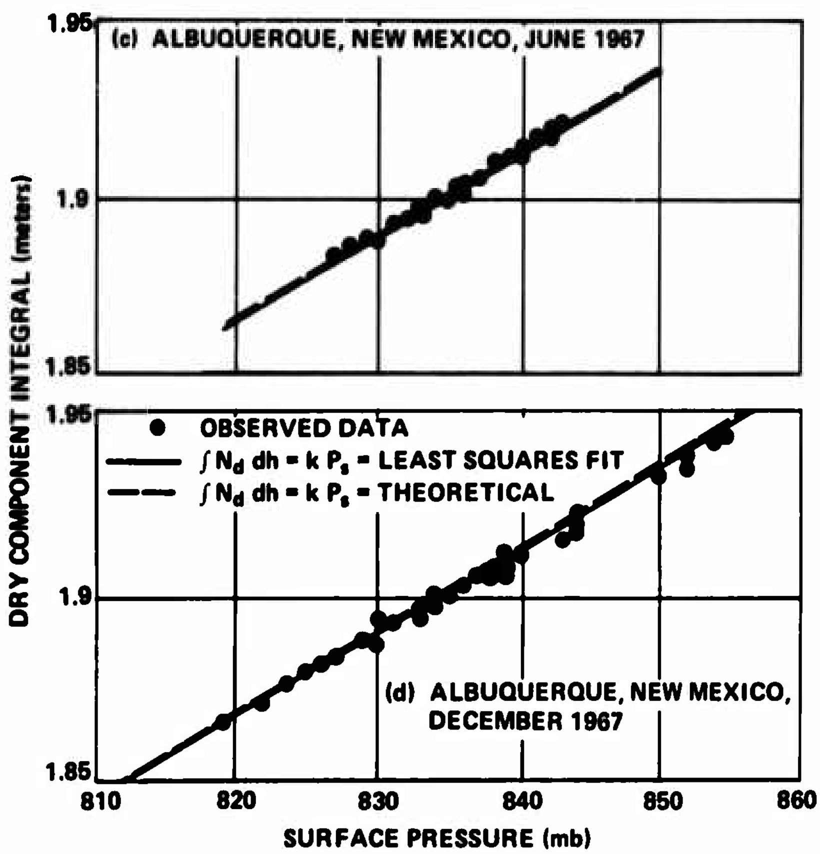

Fig. 5 (Continued)

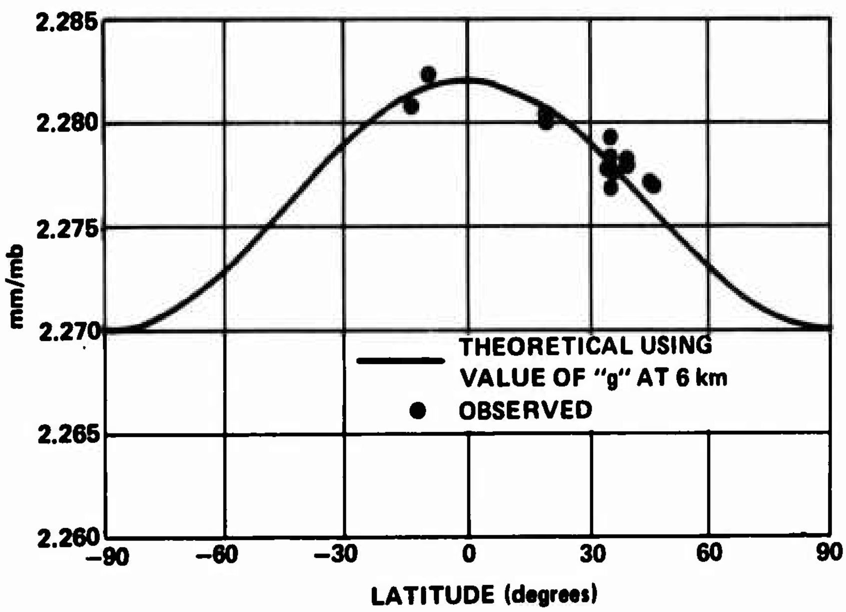

Fig. 6 ZENITH RANGE ERROR PER MILLIBAR OF SURFACE PRESSURE, DRY COMPONENT OF TROPOSPHERE

same graph are the empirical slopes k obtained for the various data sets; these also show a latitude variation and their trend is in good agreement with the theoretical variation. The empirical slope k for Albuquerque (1620 meters high) is not markedly different from the empirical slopes for the stations near sea level.

It is not certain that no biases are present, but it appears that they must be small. The data heights used in the numerical integration are geopotential heights derived from the gas laws, but these differ very little from geometric heights, especially at middle latitudes. The use of a mean (mid-atmosphere) value of g to get the theoretical slope of the line of Eq. (5) is, of course, an approximation. Water vapor content was neglected as regards the value of the gas constant per gram of air, but this effect is very small. The value used was $R = 2.8704 \times 10^6$ erg $g^{-1}{}^\circ K^{-1}$ (Ref. 11).

The height integrals $\int N_d \, dh$ have been referred to as "zenith integrals," but this is something of a misnomer. Balloons are blown by the wind and do not actually rise vertically. The height specified is the height co-ordinate of the balloon position, its lateral displacement being unknown. The fact that the integral of N_d with the height component of the oblique rise of the balloon is so closely related to pressure at the launch site would seem to argue that the horizontal gradients involved are not very large.

HEIGHT PARAMETERS FOR THE QUARTIC N_d MODEL

The prediction of only the height integral of N may not provide enough information for a range correction at larger zenith angles, where the temperature profile does have a significant effect. For this, a model of the N profile may also be needed.

Since N_d is proportional to P/T, but its height integral is proportional to surface pressure alone, it follows from Eq. (14) that the equivalent height h_d for the quartic N_d profile must be proportional to surface temperature. To

examine this theory, it was assumed that:

$$h_d = h_{d_0} + a_d T_{sC} , \tag{16}$$

where h_d is the "equivalent height" of the quartic N_d profile which will match the theoretical height integral of N_d to the observed one; h_{d_0} is the value of h_d when the surface temperature is 0°C, T_{sC} is the observed surface temperature in °C, and a is the temperature coefficient of the height increase. The parameters h_{d_0} and a_d were obtained from the data by a least-squares procedure.

To do this, Eq. (16) was combined with Eq. (14) to get a theoretical expression for $\int N_d \, dh$ corresponding to the surface N_d at the time of a balloon flight; these theoretical integrals based on the observed values of N_s were then equated to the observed integrals from balloon data. The results are listed in Table 2. The heights h_{d_o} as listed are heights above the station in each case. These were found to be closely the same for all stations regardless of the station height above sea level, and are in close agreement with the value predicted by theory (T_s/α, quoted above). These heights should be interpreted as parameters for matching observed integrals, not as indicating any undulation of actual pressure levels above surface undulations. The results may contain information not as yet deciphered, about latitude, longitude, and surface height effects. The balloons at all stations were launched at 0 and 12 hours U. T., hence at different values of local time.

Extrapolating from the empirical values of Table 2, the equivalent height of the model atmosphere would in each case fall to zero within a few degrees of 0°K. The deviations from absolute zero may be related to second-order effects (e. g., gravity anomalies, atmospheric tides, etc.). These have not yet been investigated.

Table 2

Height Parameters for the Quartic N_d Model (cf. Eq. 16)

Station	Year	h_{d_o} Above Station (km)	a_d (km/°C)	$\int N_d\,dh$, Observed Mean (meters)	Prediction Error in $\int N_d\,dh$ σ (meters)
Weather Ship E	1963	39.998	0.15111	2.32085	0.00141
Weather Ship E	1965	40.035	0.14910	2.31787	0.00144
Weather Ship E	1967	40.008	0.14976	2.32390	0.00170
Ascension Island	1967	40.099	0.14982	2.29241	0.00083
Caribou, Maine	1967	40.066	0.14830	2.25882	0.00155
Washington, D. C. (Dulles Airport)	1967	40.066	0.14893	2.29636	0.00159
St. Cloud, Minn.	1967	40.069	0.14777	2.22478	0.00128
Columbia, Mo.	1967	40.069	0.14852	2.25030	0.00189
Albuquerque, New Mexico	1967	40.057	0.14764	1.90810	0.00138
Vandenberg Air Force Base, California	1967	40.054	0.14750	2.28651	0.00147
Wake Island	1963	39.999	0.15063	2.30960	0.00156
Wake Island	1965	40.024	0.15087	2.31234	0.00154
Wake Island	1967	39.999	0.15179	2.31013	0.00187
Pago Pago, Samoa	1967	40.222	0.14409	2.30569	0.001675

The integral of N_d through the atmosphere can be predicted at the zenith or in a different direction by using the parameters of Table 2 and the expressions developed in Refs. 4, 5, and 10. Figures 7 and 8 show samples of predicting the zenith range error (dry part) from the quartic profile and the parameters of Table 2. The rms values of the deviations are less than 2 mm in all cases.

When large zenith angles are to be used, study of the temperature profile and its time variations at a given location can be of use for developing an N_d profile model to fit the location. The height parameters for a theoretical model based on any desired constant lapse rate are easily deduced from the results of Table 2 and Eqs. (11) and (12). To match a given observed integral, the equivalent height h_d of the theoretical integral varies $(\mu + 1)^{-1}$, hence the equivalent height at 0°C for a theoretical h_d profile of degree 5 (to fit a lapse rate of 4.8°C/km, cf. Fig. 1) would be (6/5) . 40 or 48 km. The temperature coefficient of h_d must then change by the same factor. Ray tracing studies may also be needed, to compare path curvature effects for different temperature profiles.

The larger the zenith angle, the more important are the effects of horizontal gradients in the atmosphere. So far, these have been neglected.

ZENITH INTEGRAL OF N_w

The integral of N_w (radio value) will be mentioned only briefly. For convenience, the N_w profile can be represented by an expression of the same degree as the N_d profile, but it does not have the same theoretical basis. The equivalent height h_w is much less than h_d, and so far, the integral of N_w is predicted from surface data with an order of magnitude less precision than the dry integral, i.e., to a few centimeters instead of millimeters. Preliminary work on N_w was reported earlier (Ref. 4), and work is continuing on this aspect of the problem. It is important for radio applications, though not for lasers.

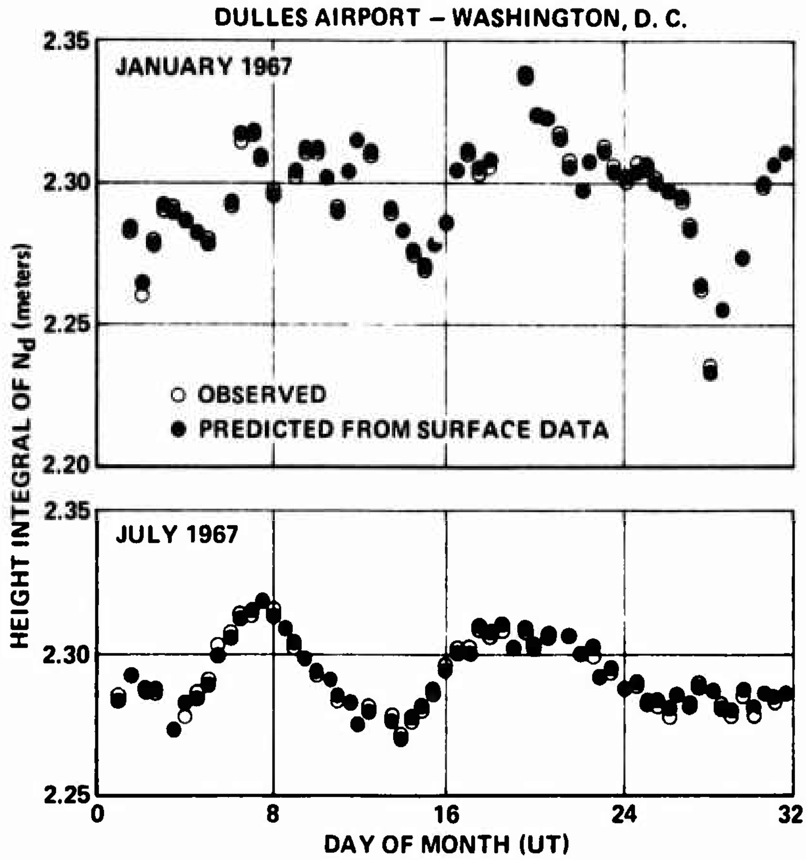

Fig. 7 OBSERVED AND PREDICTED VERTICAL INTEGRAL OF REFRACTIVITY AT WASHINGTON, D. C., JANUARY AND JULY 1967, DRY COMPONENT

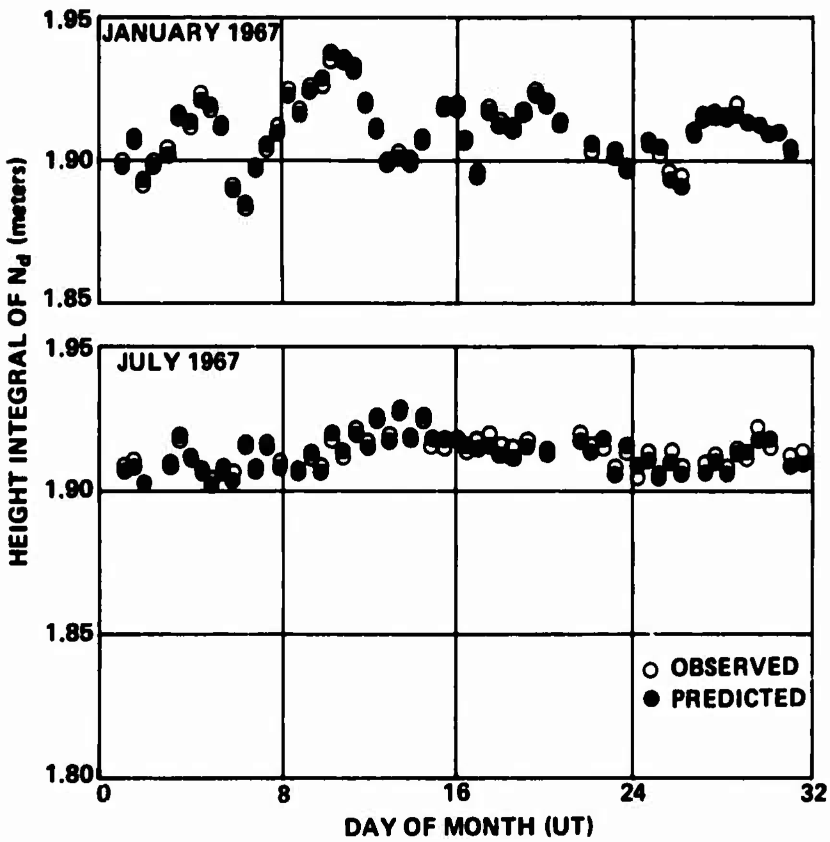

Fig. 8 OBSERVED AND PREDICTED VERTICAL INTEGRAL OF REFRACTIVITY AT ALBUQUERQUE, NEW MEXICO, JANUARY AND JULY 1967, DRY COMPONENT

REFERENCES

1. E. K. Smith, Jr. and S. Weintraub, "The Constants in the Equation for Atmospheric Refractive Index at Radio Frequencies," Proc. IRE, Vol. 41, No. 8, August 1953, p. 1035.

2. B. Edlén, "The Dispersion of Standard Air," J. Opt. Soc. Am., Vol. 43, No. 5, May 1953, p. 339.

3. E. Wood and M. C. Thompson, Jr., "The Group Refractive Index of Air," Appl. Optics, Vol. 7, No. 7, July 1968, p. 1408.

4. H. S. Hopfield, "Two-Quartic Tropospheric Refractivity Profile for Correcting Satellite Data," J. Geophys. Res., Vol. 74, No. 18, 20 August 1969, p. 4487.

5. H. S. Hopfield, "Tropospheric Effect on Electromagnetically Measured Range: Prediction from Surface Weather Data," Radio Science, Vol. 6, No. 3, March 1971, p. 357.

6. B. Haurwitz, Dynamic Meteorology, McGraw-Hill, New York, 1941.

7. B. R. Bean and E. J. Dutton, CRPL Exponential Reference Atmosphere, U. S. National Bureau of Standards Monograph No. 4, 1959.

8. B. R. Bean, "Concerning the Bi-exponential Nature of the Tropospheric Radio Refractive Index," Beitr. Phys. Atmos., Vol. 34, 1961, p. 81.

9. B. R. Bean and E. J. Dutton, Radio Meteorology, U. S. National Bureau of Standards Monograph 92, 1966.

10. S. M. Yionoulis, "Algorithm to Compute Tropospheric Refraction Effects on Range Measurements," J. Geophys. Res., Vol. 75, No. 36, 20 December 1970, p. 7636.

11. Smithsonian Meteorological Tables, prepared by R. J. List, 6th Rev. Ed., 1963.

ACKNOWLEDGMENT

The contribution of H. K. Utterback of APL, who prepared the computing programs and handled the computations, is acknowledged with appreciation. The meteorological balloon data for the study were obtained from the U. S. National Climatic Center (formerly Weather Records Center), and I wish to thank its members for their cooperation.